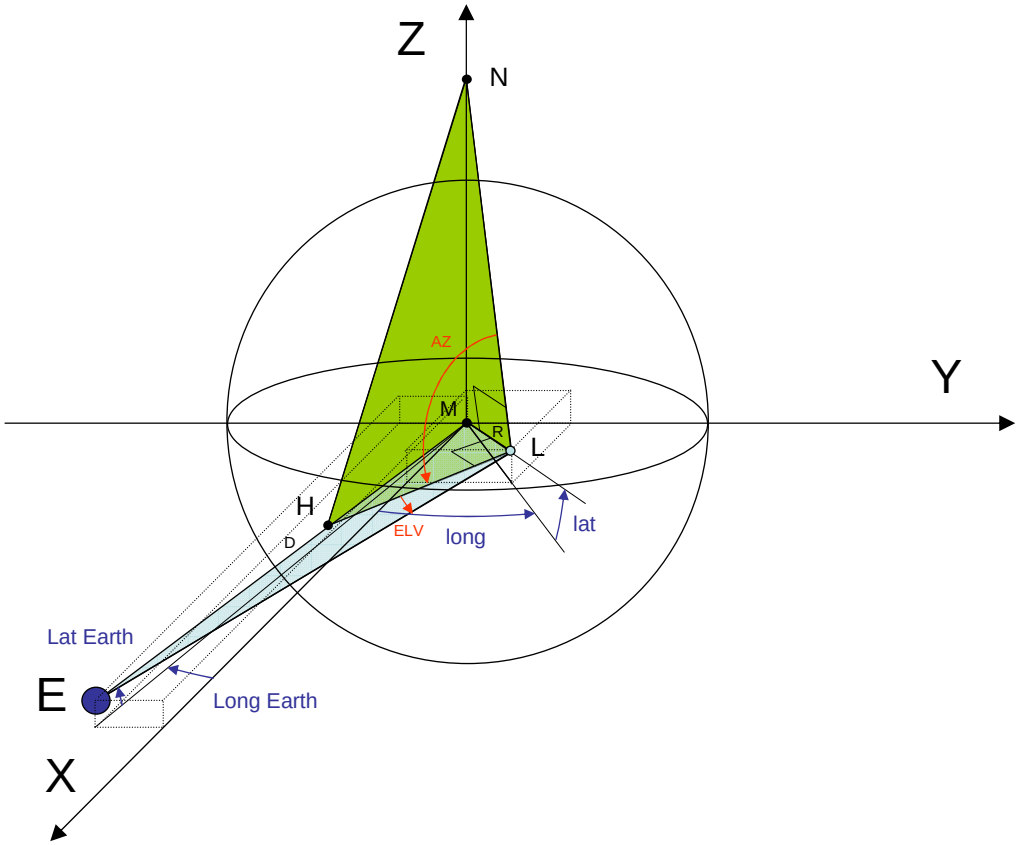
Elevation and Azimuth of the LM S-band antenna

Once on the surface of the Moon, communication with the Earth depended on the S-band antenna (around 2 GHz). The LM had three S-band antennas, two omni-directional antennas on the aft and forward sections of the ascent stage and a high gain steerable antenna on the mid-section of the ascent stage. If we want to accurately depict an Apollo landing site, it is important to orient the steerable S-band antenna correctly. The Earth, Moon and LM geometry can be shown in the following figure:



On the figure the various points represent the following items :

- M = the Moon
- E = the Earth
- L = the LM
- N = the intersection between the Horizontal plane NLH and the North direction (X axis)
- H = the intersection between the Horizontal plane NLH and the direction of the Earth

Calculating the elevation and azimuth of the antenna means to know the position of each point and their relationship with the triangles MEL and NLH. The first thing to do is to calculate the Cartesian coordinates of each point.

The Cartesian coordinates of points L, E and M are as follows :

$$L\begin{pmatrix}\cos(long)*\cos(lat)*R\\ \sin(long)*\cos(lat)*R\\ \sin(lat)*R\end{pmatrix} \quad E\begin{pmatrix}\cos(long\ Earth)*\cos(lat\ Earth)*D\\ \sin(long\ Earth)*\cos(lat\ Earth)*D\\ \sin(lat\ Earth)*D\end{pmatrix} \quad M\begin{pmatrix}0\\0\\0\end{pmatrix}$$

long and *lat* are the coordinates of the lunar module on the surface of the moon. *long Earth* and *lat Earth* are the coordinates of the sub-Earth point on the surface of the Moon. Because of Lunar Libration the Earth is not directly over the 0° Longitude and 0° Latitude of the Moon. The Earth moves in a "box" approximately 16° (8° East to 8° West) by 12° (6° North and 6° South) . The libration is constantly changing because of the

Moon's orbital motion, so the position of the Earth above the Moon at, say, landing is different than during EVA, and again different at liftoff from the Moon.

For instance, when Apollo 11 landed the Earth was directly over Longitude 7.2° W and Latitude 1.8° N. By Armstrong's first step these were 7.3° W and 2.1° N . As you can see the deviation from 0° is significa nt and needs to be taken into account in the calculation of the elevation and azimuth angles of the S-band antenna.

R is the diameter of the Moon (1738 km) and D the distance between the Earth and the Moon, this distance varies because the lunar orbit is elliptical. One way to know that distance is to know the apparent diameter of the Moon in Radian. The distance D can then be calculated as :

$$D = (2 * R) / Moon\,apparent\,diameter\,(in\,radian)$$

The coordinates of the sub-Earth point and the Moon apparent diameter can be calculated on the US Naval Observatory's web site at the following address:

http://aa.usno.navy.mil/data/docs/diskmap.php#notes

Select the Moon as the object and the date for which you want the information, then you will see an image of the Moon for the corresponding date, the sub-Earth coordinates and the apparent diameter in seconds. Please note that the field in which the lunar disk appears is oriented with north up and east to the left. These are directions on the celestial sphere that means that an East coordinate (positive) on the celestial sphere will be a West coordinate on the Lunar sphere. **So it is important to change the sign of the longitude to get a lunar longitude**.

The values of the LME triangle sides are then :

$$LM = \sqrt{L_x{}^2 + L_y{}^2 + L_z{}^2} \qquad LE = \sqrt{(L_x - E_x)^2 + (L_y - E_y)^2 + (L_z - E_z)^2} \qquad ME = D$$

Knowing the relationship between the sides of a triangle , $ME^2 = LM^2 + LE^2 - 2*LM*LE*\cos(MLE)$, one can calculate the elevation angle :

$$Elevation = \arccos\left(\frac{ME^2 - LM^2 - LE^2}{-2 * LM * LE}\right) - 90°$$

Using the same triangle relationship one can calculate the distances MH and MN

$$\cos E\hat{M}L = \frac{LE^2 - ML^2 - ME^2}{-2 * ML * ME} \qquad MH = \frac{R}{\cos E\hat{M}L} \qquad MN = \frac{R}{\sin(lat)}$$

The coordinates of N and H can then be written as :

$$N\begin{pmatrix} 0 \\ 0 \\ \frac{R}{\sin(lat)} \end{pmatrix} \qquad H\begin{pmatrix} E_x * \frac{MH}{D} \\ E_y * \frac{MH}{D} \\ E_z * \frac{MH}{D} \end{pmatrix}$$

The sides of the LHN triangles are then :

$$LH = \sqrt{(L_x - H_x)^2 + (L_y - H_y)^2 + (L_z - H_z)^2}$$

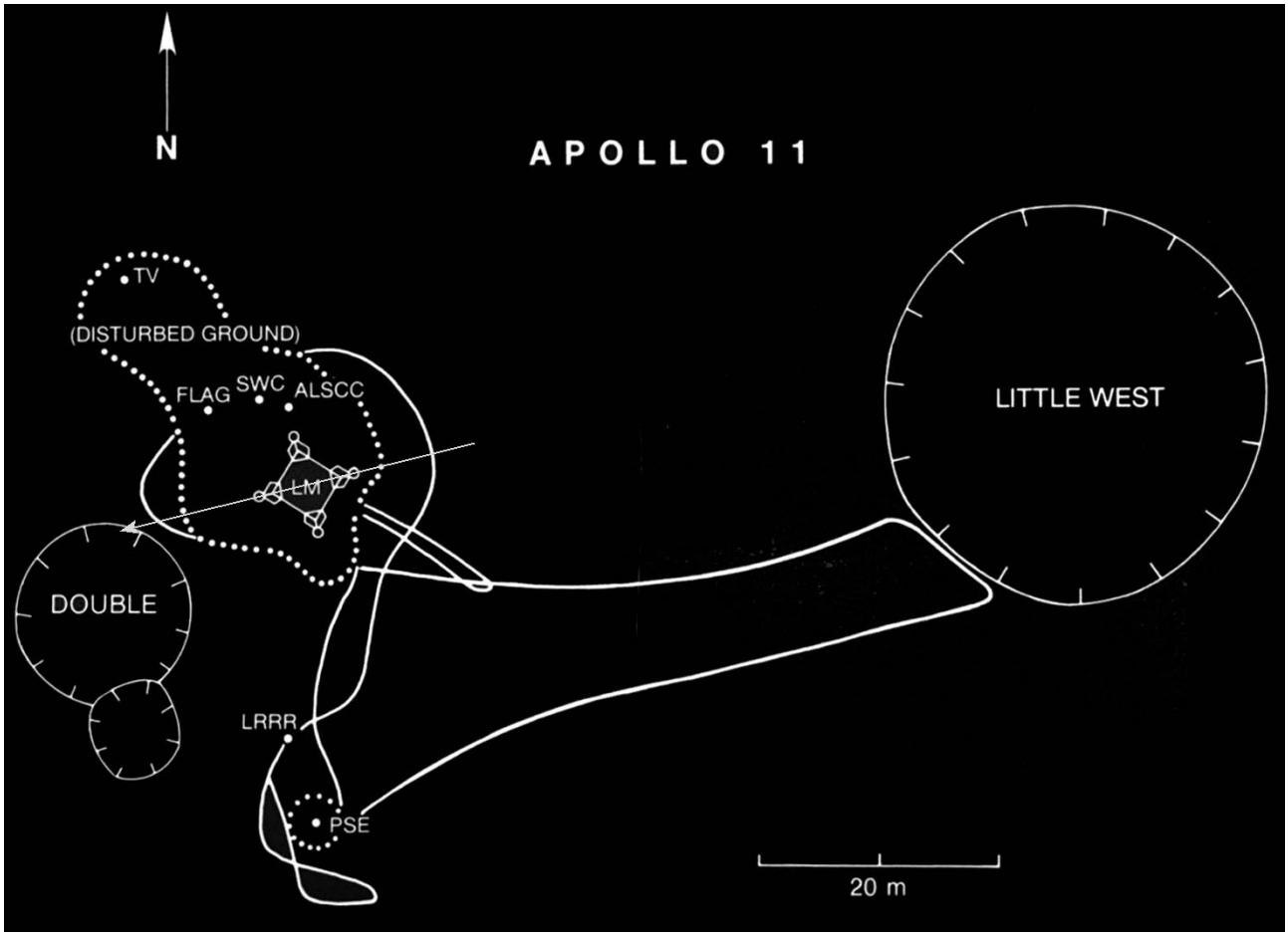
$$LN = \sqrt{L_x{}^2 + L_y{}^2 + (L_z - N_z)^2}$$

$$HN = \sqrt{H_x{}^2 + H_y{}^2 + (H_z - N_z)^2}$$

Again, knowing the relationship between the sides of a triangle , $HN^2 = HL^2 + LN^2 - 2*HL*LN*\cos(AZT)$ one can calculate the azimuth angle :

$$Azimuth = \arccos\left(\frac{HN^2 - HL^2 - LN^2}{-2 * HL * LN}\right)$$

Knowing the azimuth and elevation of the antenna is one thing but in order to appropriately render the configuration of the antenna on the landing site one has to know the azimuth of the LM and the tilt angle if any. For LM-5, the Z axis had an azimuth of about 257° relative to the North as shown on the following figure.

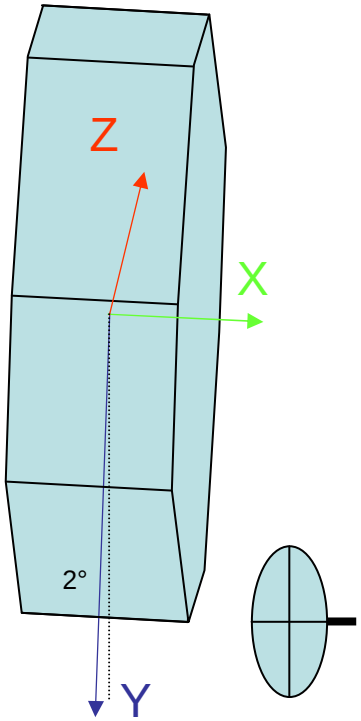


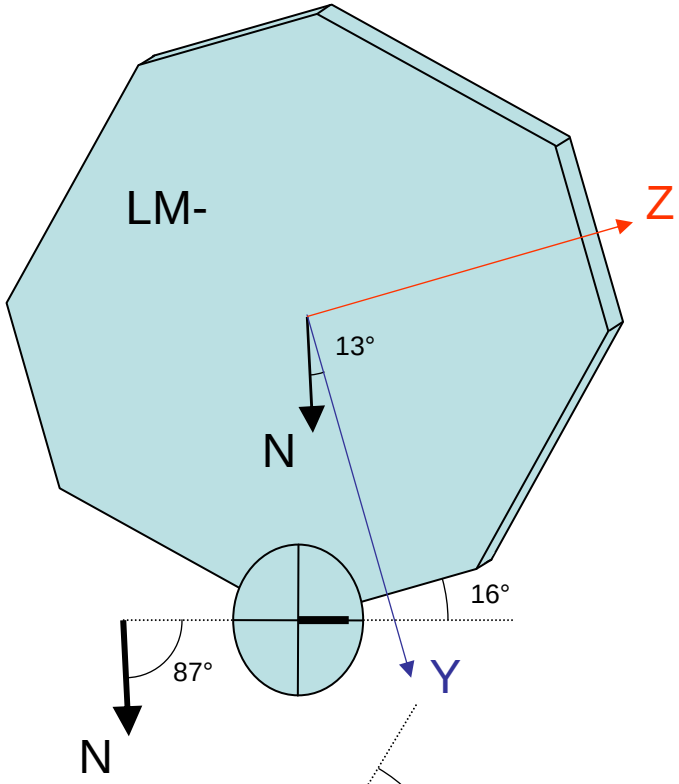
Furthermore the LM-5 pitched up about 4.5° and yawe d to the right by about 2°. Roll, Pitch and Yaw mus t be taken into account in the orientation of the S-band antenna relative to the LM itself.

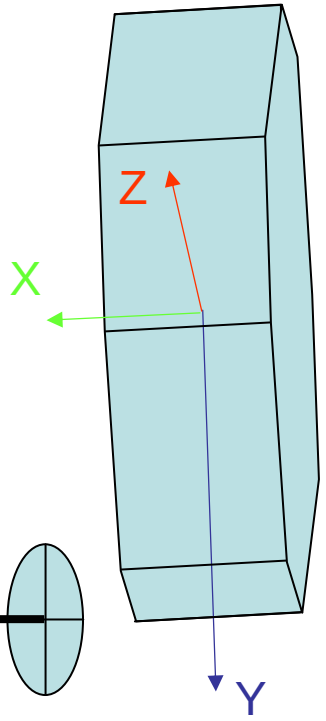
The following figures show the orientation of the S-band antenna, first with the position of the LM relative to the antenna and then the position of the antenna relative to the LM. The Tables present the calculation of antenna elevations and azimuths for each flight at the start of each EVA. Please note that the calculation for Apollo 16 is irrelevant since the steerable motor failed and the antenna could not be oriented properly towards the Earth (Astronauts used the omnidirectional S-band antennas and the LRV steerable S-band antenna during EVAs).

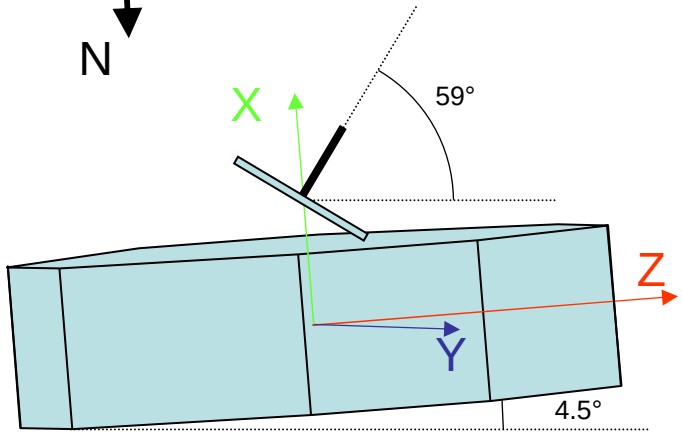
Finally the last figure shows the relative azimuth of the Earth for each flight where the tip of the arrows represents the sub-Earth point on the surface of the Moon.

Many thanks to Ed Ed Kotapish who provided me with information about the US Naval Observatory's web site and the sub-Earth point and Paul Fjeld with whom I could check my calculations were correct.





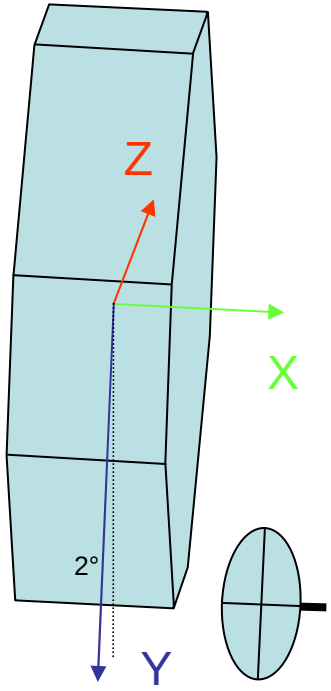


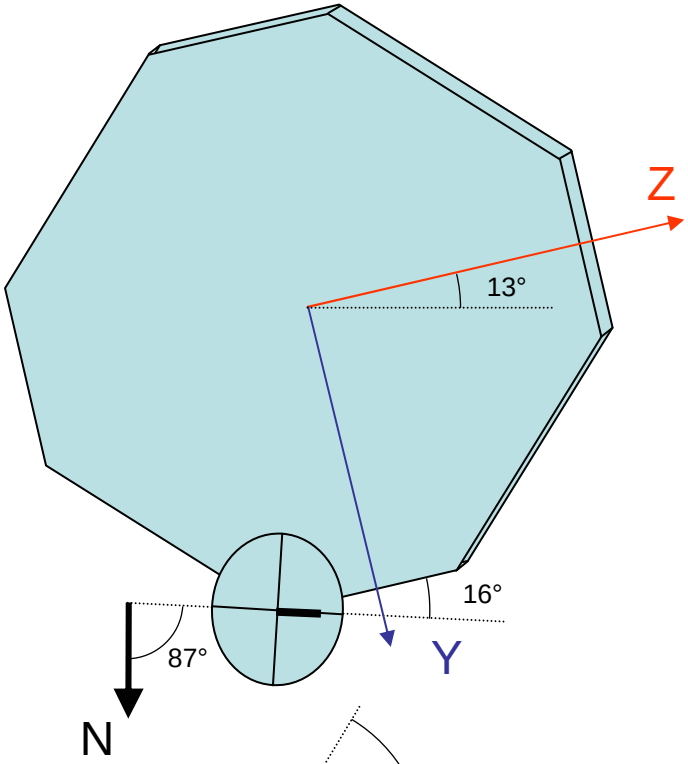


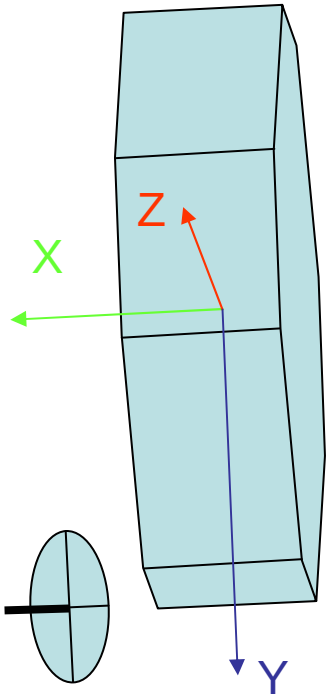
Apollo 11 – Eagle

0.67408°N
23.47297°E

Roll : +13°
Pitch : +4.5°
Yaw : +2°



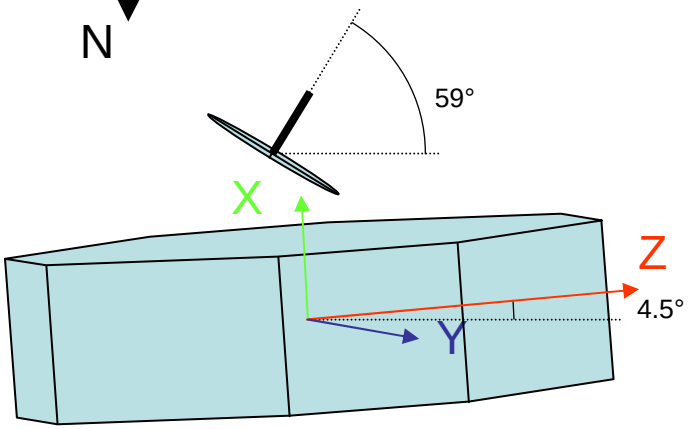


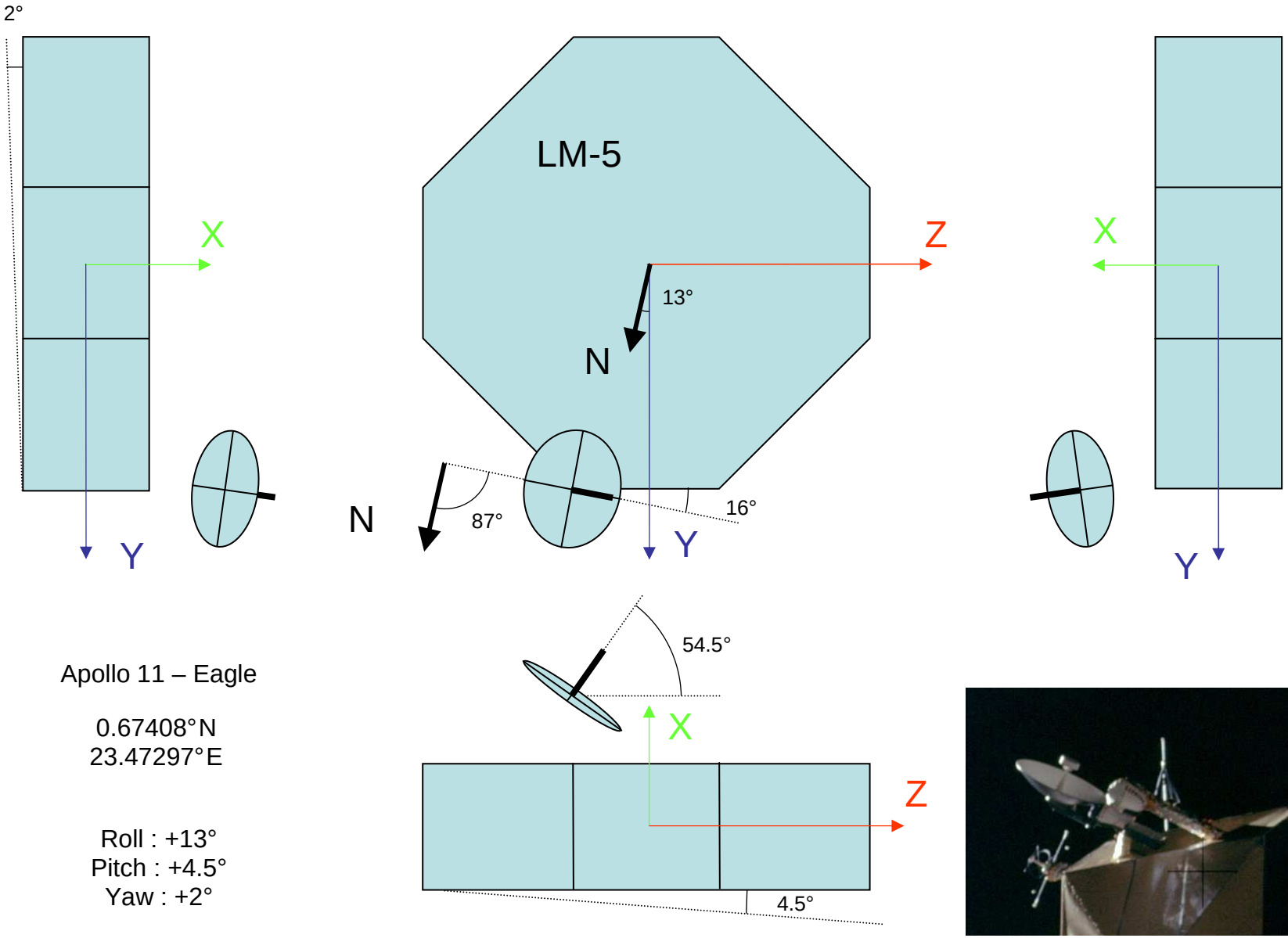


Apollo 11 – Eagle

0.67408°N
23.47297°E

Roll : +13°
Pitch : +4.5°
Yaw : +2°





Apollo 11 – Eagle

0.67408°N
23.47297°E

Roll : +13°
Pitch : +4.5°
Yaw : +2°

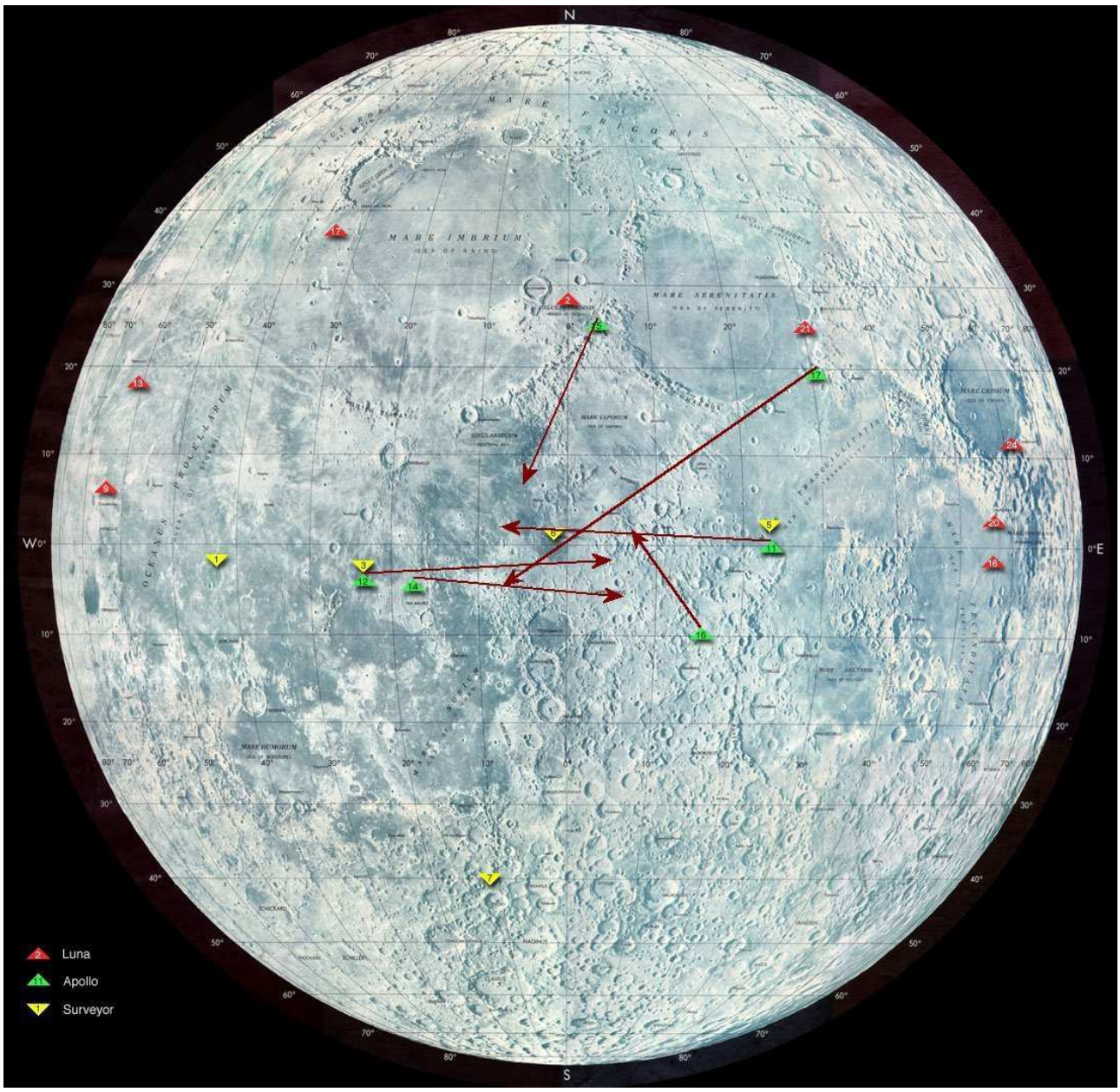
	Apollo 11 - EVA		Apollo 12 - EVA 1		Apollo 12 - EVA 2	
Long LM	**23.47297**		**-23.42157**		**-23.42157**	
Lat LM	**0.67408**		**-3.01239**		**-3.01239**	
Long Earth	**-7.3**		**5.1**		**5.2**	
Lat Earth	**2.1**		**-1.7**		**-2.7**	
Diameter	**1846.7**	"	**1886.4**	"	**1875.8**	"
D	388247.3962	km	380076.5832	km	382224.3664	km
R	1738	km	1738	km	1738	km
Lx	1594.066849	km	1592.593871	km	1592.593871	km
Ly	692.2259765	km	-689.8888607	km	-689.8888607	km
Lz	20.44695131	km	-91.33521081	km	-91.33521081	km
Ex	384841.7988	km	378405.2603	km	380228.7128	km
Ey	-49299.37142	km	33771.76977	km	34603.52361	km
Ez	14226.82449	km	-11275.44469	km	-18005.23328	km
LM	1738	km	1738	km	1738	km
LE	386755.4848	km	378550.493	km	380699.1506	km
ME	388247.3962	km	380076.5832	km	382224.3664	km
OLT	149.0722167		151.3478506		151.2879111	
Elevation	***59.07221673***	looking W	***61.34785062***	looking E	***61.28791115***	looking E
cos(EML)	0.858995997		0.878596138		0.878092158	
MH	2023.292315	km	1978.155748	km	1979.291108	km
Hx	2005.544561	km	1969.45714	km	1968.956918	km
Hy	-256.9161836	km	175.7693671	km	179.1891167	km
Hz	74.14093422	km	-58.68445126	km	-93.23737903	km
Nz	147730.7768	km	-33072.06469	km	-33072.06469	km
HL	1035.889855	km	944.6989801	km	947.0740686	km
LN	147720.553	km	33026.36551	km	33026.36551	km
HN	147670.4789	km	33072.54043	km	33038.03794	km
Azimuth	***272.9713884***		***88.01660259***		***90.11523607***	

	Apollo 14 - EVA 1		Apollo 14 - EVA 2	
Long LM	**-17.47136**		**-17.47136**	
Lat LM	**-3.6453**		**-3.6453**	
Long Earth	**6.4**		**6**	
Lat Earth	**-5.9**		**-5.3**	
Diameter	**1837.8**	"	**1825.1**	"
D	390127.58	km	392842.2917	km
R	1738	km	1738	km
Lx	1654.46694	km	1654.466945	km
Ly	-520.742346	km	-520.7423461	km
Lz	-110.501299	km	-110.5012993	km
Ex	385642.575	km	389019.9417	km
Ey	43256.7456	km	40887.64348	km
Ez	-40102.2036	km	-36287.07325	km
LM	1738	km	1738	km
LE	388539.151	km	391248.5269	km
ME	390127.58	km	392842.2917	km
OLT	156.003764		156.4422118	
Elevation	***66.0037645***	looking E	***66.44221177***	looking E
cos(EML)	0.91430747		0.917362711	
MH	1900.89228	km	1894.561419	km
Hx	1879.03914	km	1876.127363	km
Hy	210.76801	km	197.1889317	km
Hz	-195.397539	km	-175.0017512	km
Nz	-27335.8234	km	-27335.82336	km
HL	769.900931	km	754.1345827	km
LN	27280.5168	km	27280.51677	km
HN	27206.2112	km	27226.25512	km
Azimuth	***96.3437123***		***94.9164223***	

	Apollo 15 - EVA 1		Apollo 15 - EVA 2		Apollo 15 - EVA 3	
Long LM	**3.63386**		**3.63386**		**3.63386**	
Lat LM	**26.13222**		**26.13222**		**26.13222**	
Long Earth	**-4.9**		**-5.6**		**-6**	
Lat Earth	**6.8**		**6.6**		**6.1**	
Diameter	**1802.4**	"	**1821.8**	"	**1843.2**	"
D	397789.8727	km	393553.8844	km	388984.628	km
R	1738	km	1738	km	1738	km
Lx	1557.204553	km	1557.204553	km	1557.20455	km
Ly	98.89494219	km	98.89494219	km	98.8949422	km
Lz	765.4918487	km	765.4918487	km	765.491849	km
Ex	393548.0457	km	389079.883	km	384663.342	km
Ey	-33738.96911	km	-38149.61608	km	-40429.7464	km
Ez	47099.89948	km	45233.96203	km	41335.0903	km
LM	1738	km	1738	km	1738	km
LE	396167.5124	km	391936.4996	km	387374.206	km
ME	397789.8727	km	393553.8844	km	388984.628	km
OLT	158.936348		158.4822582		157.861789	
Elevation	***68.93634805***	looking W	***68.48225824***	looking W	***67.8617892***	looking W
cos(EML)	0.933744946		0.930896942		0.92691071	
MH	1861.321989	km	1867.016554	km	1875.04576	km
Hx	1841.473806	km	1845.791927	km	1854.21561	km
Hy	-157.8699946	km	-180.9814808	km	-194.885913	km
Hz	220.3879098	km	214.5895626	km	199.249996	km
Nz	3946.017198	km	3946.017198	km	3946.0172	km
HL	666.2398558	km	681.9873995	km	703.670798	km
LN	3542.655463	km	3542.655463	km	3542.65546	km
HN	4158.87753	km	4166.923845	km	4185.01619	km
Azimuth	***204.309295***		***205.8730271***		***206.321672***	

	Apollo 16 - EVA 1		Apollo 16 - EVA 2		Apollo 16 - EVA 3	
Long LM	**15.49812**		**15.49812**		**15.49812**	
Lat LM	**-8.97301**		**-8.97301**		**-8.97301**	
Long Earth	**7**		**6.3**		**5.5**	
Lat Earth	**1.8**		**3.1**		**4.2**	
Diameter	**1837.7**	"	**1816.4**	"	**1799.4**	"
D	390148.8091	km	394723.886	km	398453.077	km
R	1738	km	1738	km	1738	km
Lx	1654.308572	km	1654.308572	km	1654.30857	km
Ly	458.7219149	km	458.7219149	km	458.721915	km
Lz	-271.0744399	km	-271.0744399	km	-271.07444	km
Ex	387049.6191	km	391766.0079	km	395553.549	km
Ey	47523.71753	km	43251.36994	km	38087.4748	km
Ez	12254.87025	km	21346.19921	km	29181.985	km
LM	1738	km	1738	km	1738	km
LE	388460.498	km	393046.6323	km	396787.119	km
ME	390148.8091	km	394723.886	km	398453.077	km
OLT	166.2360613		164.7737505		163.409734	
Elevation	***76.2360613***	looking W	***74.77375052***	looking W	***73.4097336***	looking W
cos(EML)	0.97153584		0.965199332		0.95872595	
MH	1788.920108	km	1800.664322	km	1812.82252	km
Hx	1774.709625	km	1787.170977	km	1799.63069	km
Hy	217.906942	km	197.3055127	km	173.284726	km
Hz	56.19133854	km	97.37778909	km	132.767853	km
Nz	-11143.22693	km	-11143.22693	km	-11143.2269	km
HL	423.7819656	km	470.9012623	km	515.443017	km
LN	11006.85525	km	11006.85525	km	11006.8553	km
HN	11341.25422	km	11383.5014	km	11420.0156	km
Azimuth	***321.427362***		***322.385455***		***322.484909***	

	Apollo 17 - EVA 1		Apollo 17 - EVA 2		Apollo 17 - EVA 3	
Long LM	**30.77168**		**30.77168**		**30.77168**	
Lat LM	**20.1908**		**20.1908**		**20.1908**	
Long Earth	**-7.2**		**-7.4**		**-7.3**	
Lat Earth	**-4.5**		**-5.5**		**-6.2**	
Diameter	**1841.5**	"	**1864.7**	"	**1889.8**	"
D	389343.723	km	384499.633	km	379392.775	km
R	1738	km	1738	km	1738	km
Lx	1401.54566	km	1401.54566	km	1401.54566	km
Ly	834.550245	km	834.550245	km	834.550245	km
Lz	599.866356	km	599.866356	km	599.866356	km
Ex	385082.879	km	379541.782	km	374116.49	km
Ey	-48647.2808	km	-49293.8709	km	-47925.4277	km
Ez	-30547.5565	km	-36852.6567	km	-40974.1753	km
LM	1738	km	1738	km	1738	km
LE	388110.782	km	383282.648	km	378182.98	km
ME	389343.723	km	384499.633	km	379392.775	km
OLT	135.095962		134.352132		134.019276	
Elevation	***45.0959618***	looking W	***44.3521318***	looking W	***44.0192755***	looking W
cos(EML)	0.71051105		0.70137372		0.69726546	
MH	2446.12662	km	2477.99418	km	2492.59442	km
Hx	2419.35704	km	2446.0422	km	2457.92945	km
Hy	-305.635872	km	-317.685414	km	-314.868025	km
Hz	-191.920882	km	-237.505217	km	-269.198592	km
Nz	5035.52828	km	5035.52828	km	5035.52828	km
HL	1721.30515	km	1766.29872	km	1786.72409	km
LN	4726.08729	km	4726.08729	km	4726.08729	km
HN	5768.26894	km	5821.41982	km	5854.97107	km
Azimuth	***240.652161***		***239.660466***		***238.785163***	



Relative Azimuth of the Earth for each Apollo flight. The tip of the arrow represents the sub-Earth point.