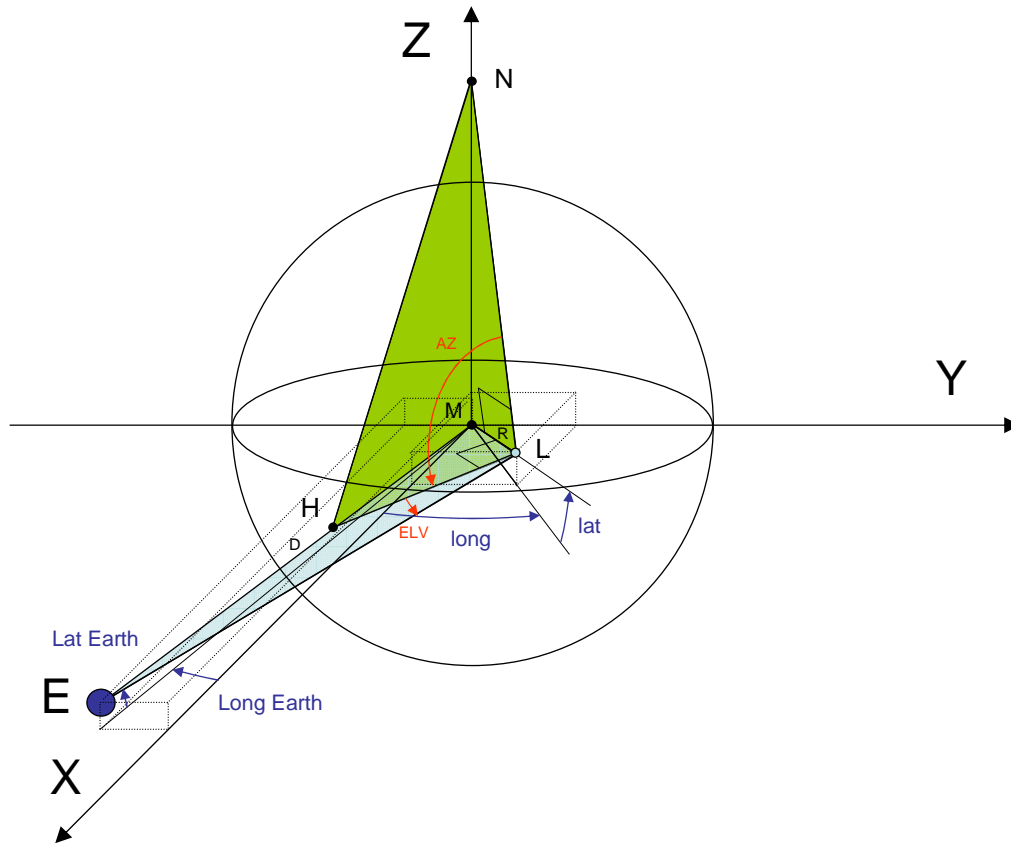


Elevation and Azimuth of the LM S-band antenna

Once on the surface of the Moon, communication with the Earth depended on the S-band antenna (around 2 GHz). The LM had three S-band antennas, two omni-directional antennas on the aft and forward sections of the ascent stage and a high gain steerable antenna on the mid-section of the ascent stage. If we want to accurately depict an Apollo landing site, it is important to orient the steerable S-band antenna correctly. The Earth, Moon and LM geometry can be shown in the following figure:



On the figure the various points represent the following items :

- M = the Moon
- E = the Earth
- L = the LM
- N = the intersection between the Horizontal plane NLH and the North direction (X axis)
- H = the intersection between the Horizontal plane NLH and the direction of the Earth

Calculating the elevation and azimuth of the antenna means to know the position of each point and their relationship with the triangles MEL and NLH. The first thing to do is to calculate the Cartesian coordinates of each point.

The Cartesian coordinates of points L, E and M are as follows :

$$L \begin{pmatrix} \cos(long) * \cos(lat) * R \\ \sin(long) * \cos(lat) * R \\ \sin(lat) * R \end{pmatrix} \quad E \begin{pmatrix} \cos(long Earth) * \cos(lat Earth) * D \\ \sin(long Earth) * \cos(lat Earth) * D \\ \sin(lat Earth) * D \end{pmatrix} \quad M \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

long and *lat* are the coordinates of the lunar module on the surface of the moon. *long Earth* and *lat Earth* are the coordinates of the sub-Earth point on the surface of the Moon. Because of Lunar Libration the Earth is not directly over the 0°Longitude and 0°Latitude of the Moon. The Earth moves in a "box" approximately 16° (8° East to 8° West) by 12° (6° North and 6° South) . The libration is constantly changing because of the

Moon's orbital motion, so the position of the Earth above the Moon at, say, landing is different than during EVA, and again different at liftoff from the Moon.

For instance, when Apollo 11 landed the Earth was directly over Longitude 7.2°W and Latitude 1.8°N. By Armstrong's first step these were 7.3°W and 2.1°N. As you can see the deviation from 0° is significant and needs to be taken into account in the calculation of the elevation and azimuth angles of the S-band antenna.

R is the diameter of the Moon (1738 km) and D the distance between the Earth and the Moon, this distance varies because the lunar orbit is elliptical. One way to know that distance is to know the apparent diameter of the Moon in Radian. The distance D can then be calculated as :

$$D = (2 * R) / \text{Moon apparent diameter (in radian)}$$

The coordinates of the sub-Earth point and the Moon apparent diameter can be calculated on the US Naval Observatory's web site at the following address:

<http://aa.usno.navy.mil/data/docs/diskmap.php#notes>

Select the Moon as the object and the date for which you want the information, then you will see an image of the Moon for the corresponding date, the sub-Earth coordinates and the apparent diameter in seconds. Please note that the field in which the lunar disk appears is oriented with north up and east to the left. These are directions on the celestial sphere that means that an East coordinate (positive) on the celestial sphere will be a West coordinate on the Lunar sphere. **So it is important to change the sign of the longitude to get a lunar longitude.**

The values of the LME triangle sides are then :

$$LM = \sqrt{L_x^2 + L_y^2 + L_z^2} \quad LE = \sqrt{(L_x - E_x)^2 + (L_y - E_y)^2 + (L_z - E_z)^2} \quad ME = D$$

Knowing the relationship between the sides of a triangle, $ME^2 = LM^2 + LE^2 - 2 * LM * LE * \cos(MLE)$, one can calculate the elevation angle :

$$\text{Elevation} = \arccos\left(\frac{ME^2 - LM^2 - LE^2}{-2 * LM * LE}\right) - 90^\circ$$

Using the same triangle relationship one can calculate the distances MH and MN

$$\cos \hat{EML} = \frac{LE^2 - ML^2 - ME^2}{-2 * ML * ME} \quad MH = \frac{R}{\cos \hat{EML}} \quad MN = \frac{R}{\sin(\text{lat})}$$

The coordinates of N and H can then be written as :

$$N \begin{pmatrix} 0 \\ 0 \\ R \\ \sin(\text{lat}) \end{pmatrix} \quad H \begin{pmatrix} E_x * \frac{MH}{D} \\ E_y * \frac{MH}{D} \\ E_z * \frac{MH}{D} \end{pmatrix}$$

The sides of the LHN triangles are then :

$$LH = \sqrt{(L_x - H_x)^2 + (L_y - H_y)^2 + (L_z - H_z)^2}$$

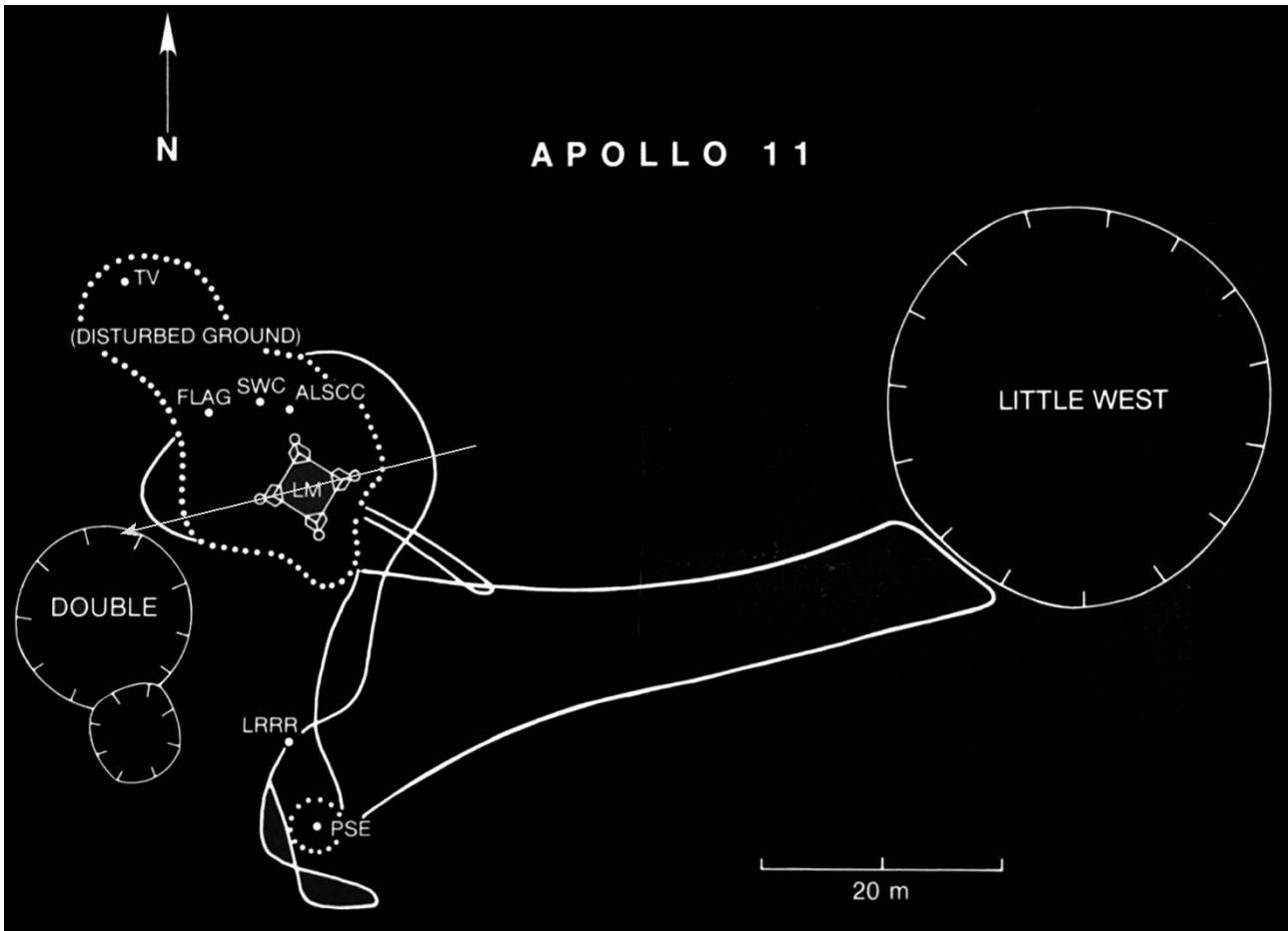
$$LN = \sqrt{L_x^2 + L_y^2 + (L_z - N_z)^2}$$

$$HN = \sqrt{H_x^2 + H_y^2 + (H_z - N_z)^2}$$

Again, knowing the relationship between the sides of a triangle, $HN^2 = HL^2 + LN^2 - 2*HL*LN*\cos(AZT)$ one can calculate the azimuth angle :

$$Azimuth = \arccos\left(\frac{HN^2 - HL^2 - LN^2}{-2 * HL * LN}\right)$$

Knowing the azimuth and elevation of the antenna is one thing but in order to appropriately render the configuration of the antenna on the landing site one has to know the azimuth of the LM and the tilt angle if any. For LM-5, the Z axis had an azimuth of about 257° relative to the North as shown on the following figure.

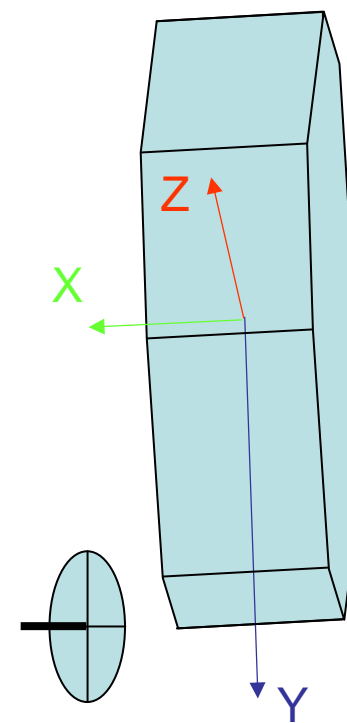
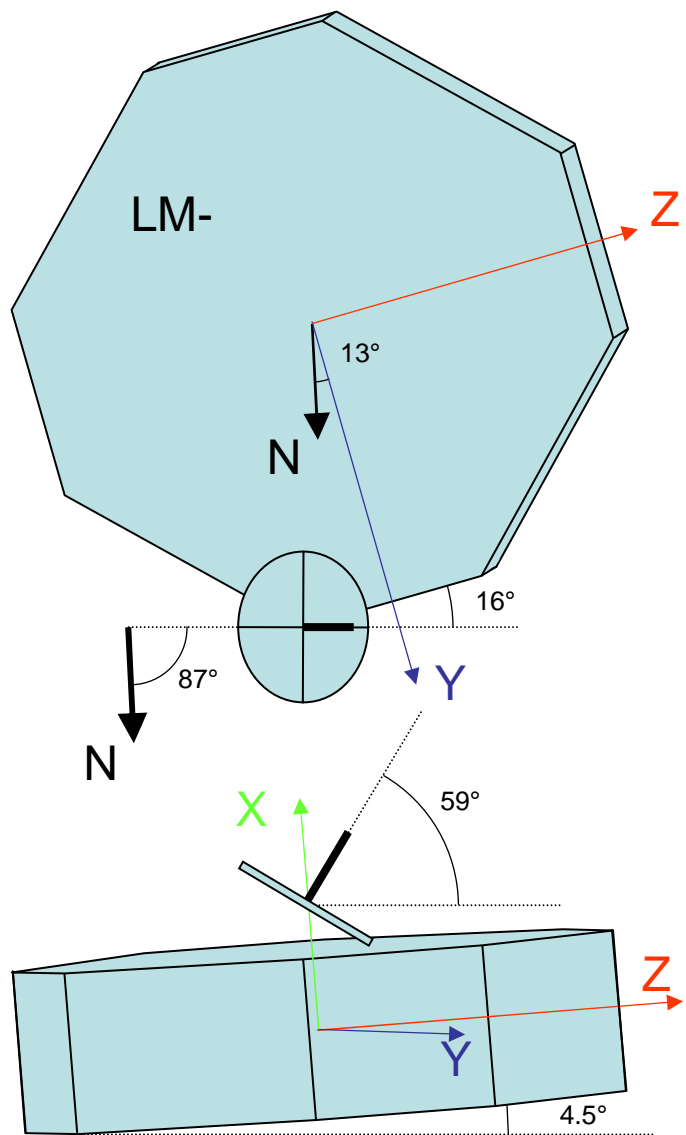
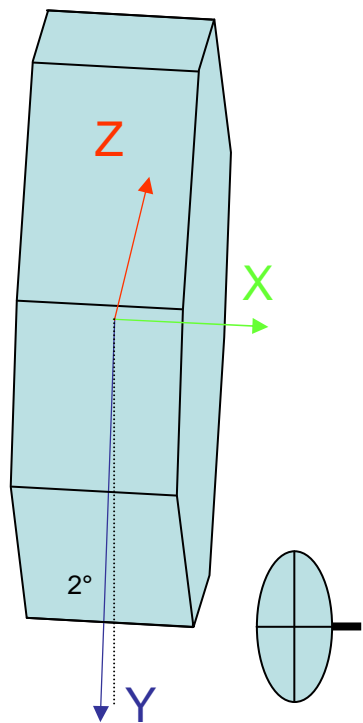


Furthermore the LM-5 pitched up about 4.5° and yawed to the right by about 2°. Roll, Pitch and Yaw must be taken into account in the orientation of the S-band antenna relative to the LM itself.

The following figures show the orientation of the S-band antenna, first with the position of the LM relative to the antenna and then the position of the antenna relative to the LM. The Tables present the calculation of antenna elevations and azimuths for each flight at the start of each EVA. Please note that the calculation for Apollo 16 is irrelevant since the steerable motor failed and the antenna could not be oriented properly towards the Earth (Astronauts used the omnidirectional S-band antennas and the LRV steerable S-band antenna during EVAs).

Finally the last figure shows the relative azimuth of the Earth for each flight where the tip of the arrows represents the sub-Earth point on the surface of the Moon.

Many thanks to Ed Ed Kotapish who provided me with information about the US Naval Observatory's web site and the sub-Earth point and Paul Fjeld with whom I could check my calculations were correct.

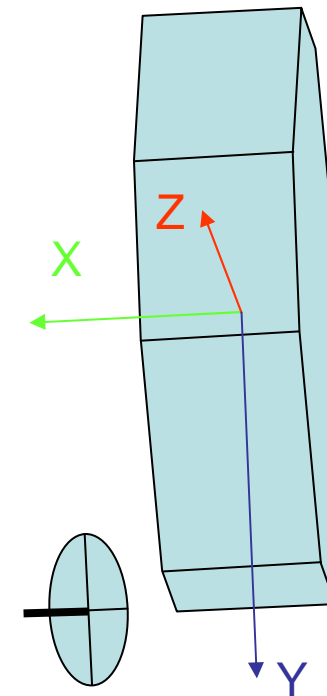
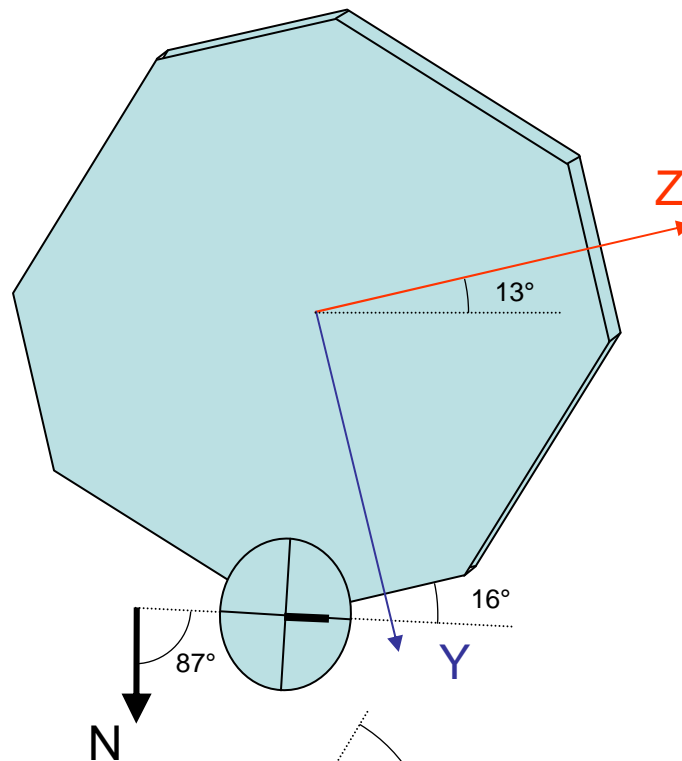
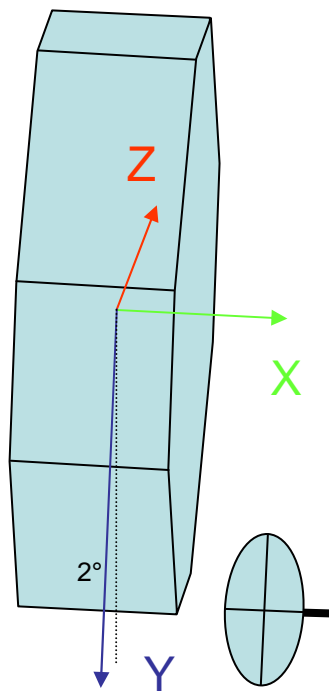


Apollo 11 – Eagle

0.67408°N
23.47297°E

Roll : +13°
Pitch : +4.5°
Yaw : +2°

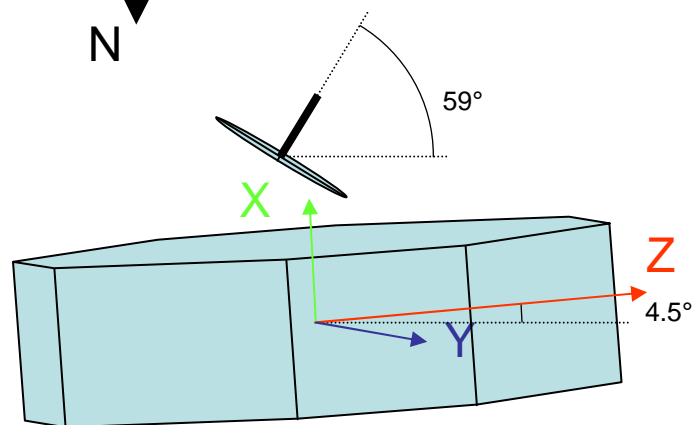


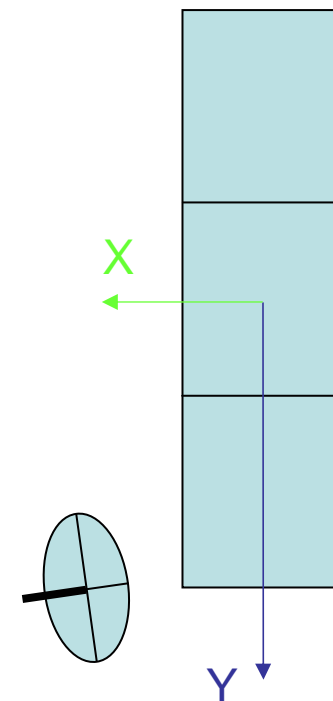
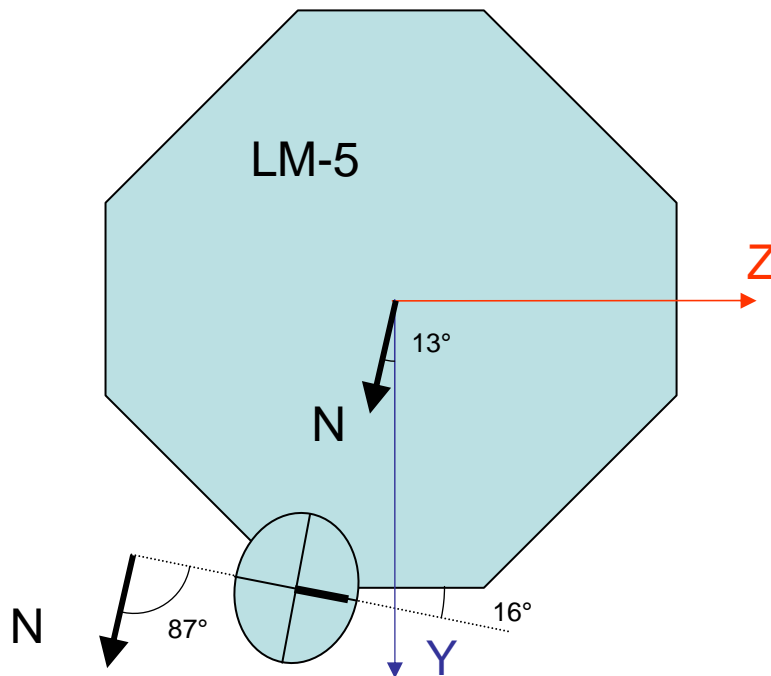
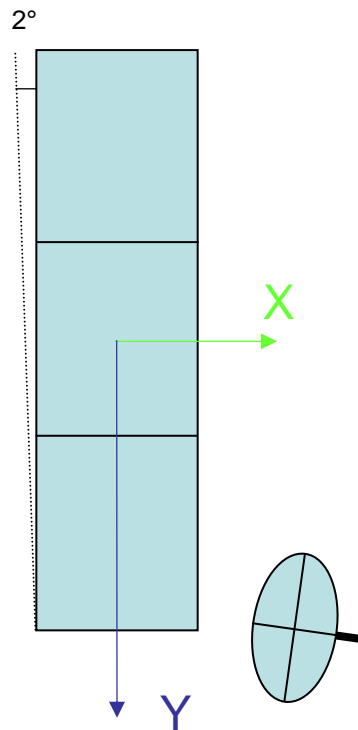


Apollo 11 – Eagle

0.67408°N
23.47297°E

Roll : +13°
Pitch : +4.5°
Yaw : +2°

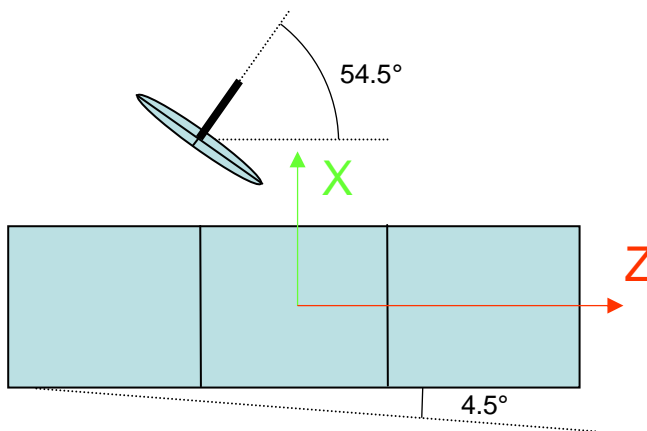




Apollo 11 – Eagle

0.67408°N
23.47297°E

Roll : +13°
Pitch : +4.5°
Yaw : +2°



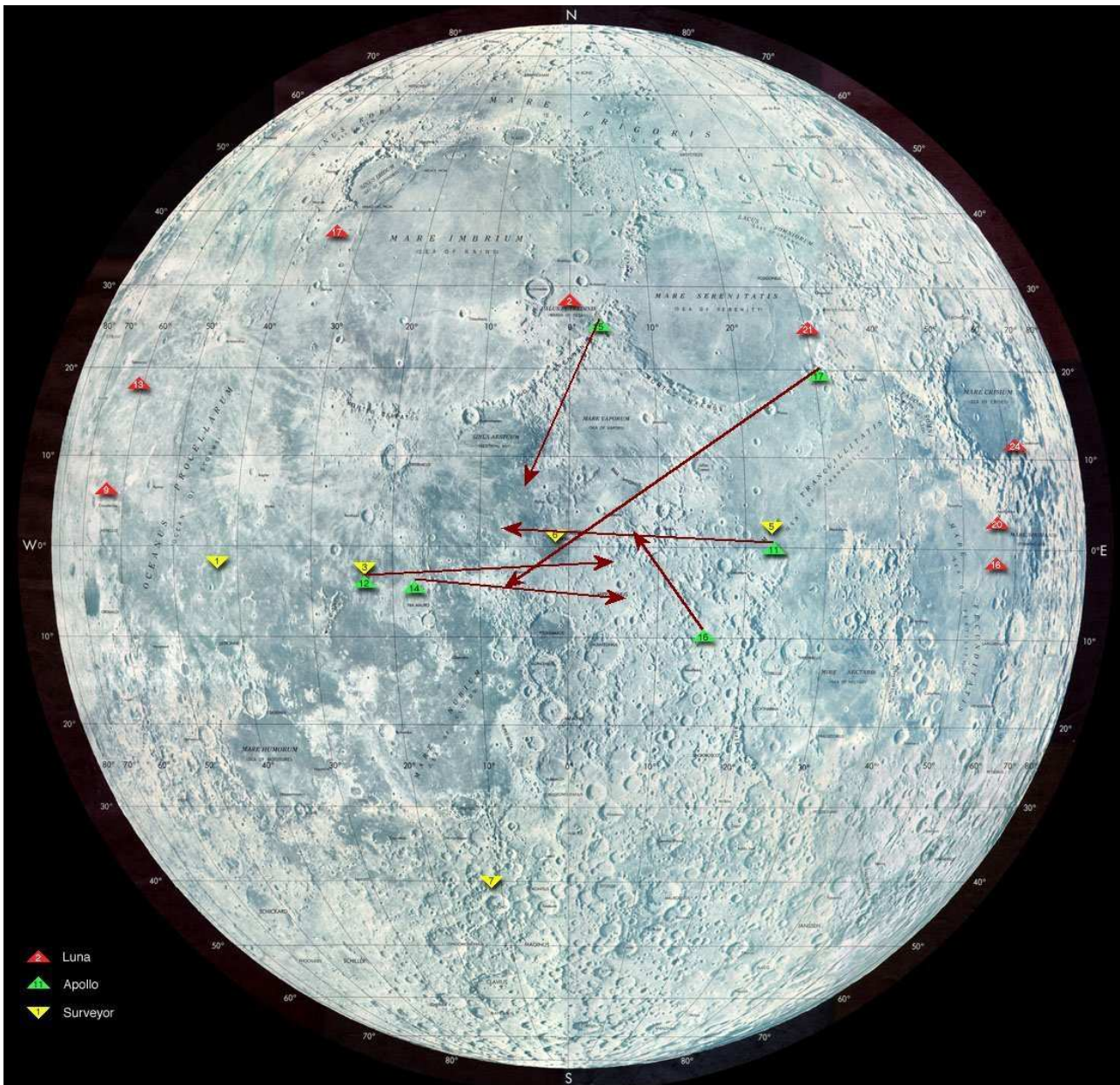
	Apollo 11 - EVA		Apollo 12 - EVA 1		Apollo 12 - EVA 2	
Long LM	23.47297		-23.42157		-23.42157	
Lat LM	0.67408		-3.01239		-3.01239	
Long Earth	-7.3		5.1		5.2	
Lat Earth	2.1		-1.7		-2.7	
Diameter	1846.7 "		1886.4 "		1875.8 "	
D	388247.3962	km	380076.5832	km	382224.3664	km
R	1738	km	1738	km	1738	km
Lx	1594.066849	km	1592.593871	km	1592.593871	km
Ly	692.2259765	km	-689.8888607	km	-689.8888607	km
Lz	20.44695131	km	-91.33521081	km	-91.33521081	km
Ex	384841.7988	km	378405.2603	km	380228.7128	km
Ey	49299.37142	km	33771.76977	km	34603.52361	km
Ez	14226.82449	km	-11275.44469	km	-18005.23328	km
LM	1738	km	1738	km	1738	km
LE	386755.4848	km	378550.493	km	380699.1506	km
ME	388247.3962	km	380076.5832	km	382224.3664	km
OLT	149.0722167		151.3478506		151.2879111	
Elevation	59.07221673	looking W	61.34785062	looking E	61.28791115	looking E
cos(EML)	0.858995997		0.878596138		0.878092158	
MH	2023.292315	km	1978.155748	km	1979.291108	km
Hx	2005.544561	km	1969.45714	km	1968.956918	km
Hy	256.9161836	km	175.7693671	km	179.1891167	km
Hz	74.14093422	km	-58.68445126	km	-93.23737903	km
Nz	147730.7768	km	-33072.06469	km	-33072.06469	km
HL	1035.889855	km	944.6989801	km	947.0740686	km
LN	147720.553	km	33026.36551	km	33026.36551	km
HN	147670.4789	km	33072.54043	km	33038.03794	km
Azimuth	272.9713884		88.01660259		90.11523607	

	Apollo 14 - EVA 1	Apollo 14 - EVA 2
Long LM	-17.47136	-17.47136
Lat LM	-3.6453	-3.6453
Long Earth	6.4	6
Lat Earth	-5.9	-5.3
Diameter	1837.8 "	1825.1 "
D	390127.58 km	392842.2917 km
R	1738 km	1738 km
Lx	1654.46694 km	1654.466945 km
Ly	-520.742346 km	-520.7423461 km
Lz	-110.501299 km	-110.5012993 km
Ex	385642.575 km	389019.9417 km
Ey	43256.7456 km	40887.64348 km
Ez	-40102.2036 km	-36287.07325 km
LM	1738 km	1738 km
LE	388539.151 km	391248.5269 km
ME	390127.58 km	392842.2917 km
OLT	156.003764	156.4422118
Elevation	66.0037645 looking E	66.44221177 looking E
cos(EML)	0.91430747	0.917362711
MH	1900.89228 km	1894.561419 km
Hx	1879.03914 km	1876.127363 km
Hy	210.76801 km	197.1889317 km
Hx	-195.397539 km	-175.0017512 km
Nz	-27335.8234 km	-27335.82336 km
HL	769.900931 km	754.1345827 km
LN	27280.5168 km	27280.51677 km
HN	27206.2112 km	27226.25512 km
Azimuth	96.3437123	94.9164223

	Apollo 15 - EVA 1	Apollo 15 - EVA 2	Apollo 15 - EVA 3
Long LM	3.63386	3.63386	3.63386
Lat LM	26.13222	26.13222	26.13222
Long Earth	-4.9	-5.6	-6
Lat Earth	6.8	6.6	6.1
Diameter	1802.4 "	1821.8 "	1843.2 "
D	397789.8727 km	393553.8844 km	388984.628 km
R	1738 km	1738 km	1738 km
Lx	1557.204553 km	1557.204553 km	1557.20455 km
Ly	98.89494219 km	98.89494219 km	98.8949422 km
Lz	765.4918487 km	765.4918487 km	765.491849 km
Ex	393548.0457 km	389079.883 km	384663.342 km
Ey	-33738.96911 km	-38149.61608 km	40429.7464 km
Ez	47099.89948 km	45233.96203 km	41335.0903 km
LM	1738 km	1738 km	1738 km
LE	396167.5124 km	391936.4996 km	387374.206 km
ME	397789.8727 km	393553.8844 km	388984.628 km
OLT	158.936348	158.4822582	157.861789
Elevation	68.93634805 looking W	68.48225824 looking W	67.8617892 looking W
cos(EML)	0.933744946	0.930896942	0.92691071
MH	1861.321989 km	1867.016554 km	1875.04576 km
Hx	1841.473806 km	1845.791927 km	1854.21561 km
Hy	-157.8699946 km	-180.9814808 km	194.885913 km
Hz	220.3879098 km	214.5895626 km	199.249996 km
Nz	3946.017198 km	3946.017198 km	3946.0172 km
HL	666.2398558 km	681.9873995 km	703.670798 km
LN	3542.655463 km	3542.655463 km	3542.65546 km
HN	4158.87753 km	4166.923845 km	4185.01619 km
Azimuth	204.309295	205.8730271	206.321672

	Apollo 16 - EVA 1			Apollo 16 - EVA 2			Apollo 16 - EVA 3		
Long LM	15.49812			15.49812			15.49812		
Lat LM	-8.97301			-8.97301			-8.97301		
Long Earth	7			6.3			5.5		
Lat Earth	1.8			3.1			4.2		
Diameter	1837.7 "			1816.4 "			1799.4 "		
D	390148.8091	km		394723.886	km		398453.077	km	
R	1738	km		1738	km		1738	km	
Lx	1654.308572	km		1654.308572	km		1654.30857	km	
Ly	458.7219149	km		458.7219149	km		458.721915	km	
Lz	-271.0744399	km		-271.0744399	km		-271.07444	km	
Ex	387049.6191	km		391766.0079	km		395553.549	km	
Ey	47523.71753	km		43251.36994	km		38087.4748	km	
Ez	12254.87025	km		21346.19921	km		29181.985	km	
LM	1738	km		1738	km		1738	km	
LE	388460.498	km		393046.6323	km		396787.119	km	
ME	390148.8091	km		394723.886	km		398453.077	km	
OLT	166.2360613			164.7737505			163.409734		
Elevation	76.2360613		looking W	74.77375052		looking W	73.4097336		looking W
cos(EML)	0.97153584			0.965199332			0.95872595		
MH	1788.920108	km		1800.664322	km		1812.82252	km	
Hx	1774.709625	km		1787.170977	km		1799.63069	km	
Hy	217.906942	km		197.3055127	km		173.284726	km	
Hx	56.19133854	km		97.37778909	km		132.767853	km	
Nz	-11143.22693	km		-11143.22693	km		-11143.2269	km	
HL	423.7819656	km		470.9012623	km		515.443017	km	
LN	11006.85525	km		11006.85525	km		11006.8553	km	
HN	11341.25422	km		11383.5014	km		11420.0156	km	
Azimuth	321.427362			322.385455			322.484909		

	Apollo 17 - EVA 1	Apollo 17 - EVA 2	Apollo 17 - EVA 3
Long LM	30.77168	30.77168	30.77168
Lat LM	20.1908	20.1908	20.1908
Long Earth	-7.2	-7.4	-7.3
Lat Earth	-4.5	-5.5	-6.2
Diameter	1841.5 "	1864.7 "	1889.8 "
D	389343.723 km	384499.633 km	379392.775 km
R	1738 km	1738 km	1738 km
Lx	1401.54566 km	1401.54566 km	1401.54566 km
Ly	834.550245 km	834.550245 km	834.550245 km
Lz	599.866356 km	599.866356 km	599.866356 km
Ex	385082.879 km	379541.782 km	374116.49 km
Ey	-48647.2808 km	49293.8709 km	47925.4277 km
Ez	-30547.5565 km	36852.6567 km	40974.1753 km
LM	1738 km	1738 km	1738 km
LE	388110.782 km	383282.648 km	378182.98 km
ME	389343.723 km	384499.633 km	379392.775 km
OLT	135.095962	134.352132	134.019276
Elevation	45.0959618 looking W	44.3521318 looking W	44.0192755 looking W
cos(EML)	0.71051105	0.70137372	0.69726546
MH	2446.12662 km	2477.99418 km	2492.59442 km
Hx	2419.35704 km	2446.0422 km	2457.92945 km
Hy	-305.635872 km	317.685414 km	314.868025 km
Hx	-191.920882 km	237.505217 km	269.198592 km
Nz	5035.52828 km	5035.52828 km	5035.52828 km
HL	1721.30515 km	1766.29872 km	1786.72409 km
LN	4726.08729 km	4726.08729 km	4726.08729 km
HN	5768.26894 km	5821.41982 km	5854.97107 km
Azimuth	240.652161	239.660466	238.785163



Relative Azimuth of the Earth for each Apollo flight. The tip of the arrow represents the sub-Earth point.